

Quark-Lepton Mass Matrices with $U(1) \times Z_2 \times Z'_2$ Flavor Symmetry

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Abstract

The $U(1)$ flavor symmetry explains the large mixing of neutrinos while it leads to the unique texture for the quark mass matrices. It is remarked that $U(1)$ symmetric mass matrices have the phenomenological defects. In the quark sector, the mixing V_{ub} is predicted to be large compared with the expected value λ^4 at the GUT scale. In the lepton sector, $U(1)$ charges, which give a large mixing in the neutrino sector, also lead to the large one in the charged lepton sector. In the viewpoint of the flavor symmetry, this is an unpleasant feature because the neutrino mass hierarchy is determined only by unknown coefficients of $\mathcal{O}(1)$, and the near-maximal flavor mixing is not guaranteed in the case of both large angle rotations. These defects disappear by introducing additional discrete symmetries $Z_2 \times Z'_2$. The $U(1) \times Z_2 \times Z'_2$ quark-lepton mass matrices are presented by taking account of the recent data of atmospheric neutrinos and solar neutrinos.

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1 Introduction

The standard model (SM) has still unexplained features such as the quark-lepton mass spectra and the flavor mixings. Mixings of the quark sector (CKM matrix) [1] seem also to have an hierarchical structure. Those features may provide an important basis for a new physics beyond the SM. On the other hand, the flavor mixing of the lepton sector, so called MNS mixing matrix [2] is still ambiguous although neutrino oscillation experiments can provide information of the fundamental property of neutrinos. In these years, there is growing experimental evidences of neutrino oscillations. The exciting one is the atmospheric neutrino deficit [3][4] as well as the solar neutrino deficit [5][6]. Super-Kamiokande [7] presented the near-maximal neutrino flavor oscillation in atmospheric neutrinos. Furthermore a new stage is represented by the long baseline(LBL) neutrino oscillation experiments [8][9][10][11] to confirm the large neutrino flavor oscillation. Since the CHOOZ result [8] excludes the large neutrino oscillation of $\nu_\mu \rightarrow \nu_e$ as far as $\Delta m^2 \geq 9 \times 10^{-4} \text{eV}^2$, the large mixing between ν_μ and ν_τ is a reasonable interpretation for the atmospheric ν_μ deficit.

It will be important to understand why there is the large flavor mixing in the lepton sector in contrast to small mixings in the quark sector. Is there a possible flavor symmetry providing a large mixing angle in the lepton mass matrices, which are consistent with the quark ones? There is a simple explanation of the large mixing by the $U(1)$ flavor symmetry [12]. The $U(1)$ flavor symmetry leads to the unique texture for the quark mass matrices [13]. However, it is remarked that $U(1)$ symmetric quark-lepton mass matrices have phenomenological defects. These defects disappear by introducing additional discrete symmetries $Z_2 \times Z'_2$. In this paper, we discuss phenomenological defects of $U(1)$ symmetric mass matrices and present $U(1) \times Z_2 \times Z'_2$ quark-lepton mass matrices.

Our approach is to assume that oscillations need only account for the atmospheric

and solar neutrino data. Since the result of LSND [14] awaits confirmation by the KARMEN experiment [15], we do not take into consideration the LSND data. Our starting point of neutrino masses and mixings is the atmospheric $\nu_\mu \rightarrow \nu_\tau$ oscillation with

$$\Delta m_{\text{atm}}^2 = 5 \times 10^{-4} - 6 \times 10^{-3} \text{eV}^2, \quad \sin^2 2\theta_{\text{atm}} \geq 0.8. \quad (1)$$

For the solar neutrino $\nu_e \rightarrow \nu_e$ oscillation, some solutions are still available [16]. In our paper, we take the small angle solution of MSW [17]:

$$\Delta m_\odot^2 \simeq 5.4 \times 10^{-6} \text{eV}^2, \quad \sin^2 2\theta_\odot \simeq 6.0 \times 10^{-3}. \quad (2)$$

Since $\Delta m_{\text{atm}}^2 \gg \Delta m_\odot^2$, we can take neutrino masses as

$$m_3 \gg m_2 \geq m_1, \quad (3)$$

or

$$m_3 \simeq m_2 \simeq m_1. \quad (4)$$

In the hierarchy case of eq.(3), the neutrino mass ratio of m_2 and m_3 is

$$\frac{m_{\nu_2}}{m_{\nu_3}} = \frac{1}{10} - \frac{1}{30} \simeq \lambda^2, \quad (5)$$

where $\lambda = 0.22$. This ratio gives a strong constraint for the Z_m symmetry [18]. On the other hand, in the quasi-degenerate case of eq.(4), the structure of the neutrino mass matrix is restricted in the framework of the $U(1)$ flavor symmetry [19].

2 Phenomenology of $U(1)$ Flavor Symmetry

We start with discussing well known regularities of the fermion mass ratios and the CKM matrix elements. The fermion mass ratios at the GUT scale are given in terms of $\lambda \simeq 0.22$ [20] as follows:

$$\frac{m_c}{m_t} \sim \lambda^4, \quad \frac{m_u}{m_t} \sim \lambda^8, \quad \frac{m_s}{m_b} \sim \lambda^2, \quad \frac{m_d}{m_b} \sim \lambda^4, \quad \frac{m_\mu}{m_\tau} \sim \lambda^2, \quad \frac{m_e}{m_\tau} \sim \lambda^4, \quad (6)$$

for the internal mass hierarchy and

$$\frac{m_b}{m_t} \sim \lambda^3, \quad \frac{m_b}{m_\tau} \sim 1, \quad (7)$$

for the intrafamily hierarchy. The CKM matrix elements at the GUT scale are

$$V_{us} = \lambda, \quad V_{cb} = 0.03 - 0.036 \simeq \lambda^2, \quad V_{ub} = 0.0015 - 0.004 \simeq \lambda^4, \quad \left| \frac{V_{ub}}{V_{cb}} \right| = \frac{1}{4}\lambda - \frac{1}{2}\lambda, \quad (8)$$

which have been derived by using RGE's of the minimal SUSY model [20]. In order to get these desirable relations of down quark masses and the CKM matrix elements, the natural down quark mass matrix M_d is expressed in the hierarchy base [13]:

$$M_d \simeq \lambda^3 m_0 \begin{pmatrix} \lambda^\delta & \lambda^3 & \lambda^\eta \\ & \lambda^2 & \lambda^2 \\ & & 1 \end{pmatrix}, \quad (9)$$

where $\delta \geq 4$ and $\eta \geq 4$. On the other hand, the up quark mass matrix M_u is

$$M_u \simeq m_0 \begin{pmatrix} \lambda^8 & & \\ & \lambda^4 & \lambda^\kappa \\ & & 1 \end{pmatrix}, \quad (10)$$

where $\kappa \geq 2$. The $U(1)$ flavor symmetry determines unknown entries. When we integrate out massive fermions the effective Yukawa couplings below the mass scale Λ are of the form [21] [22]

$$Q_i \bar{d}_j H_d \left(\frac{S}{\Lambda_d} \right)^{m_{ij}} + Q_i \bar{u}_j H_u \left(\frac{S}{\Lambda_u} \right)^{n_{ij}} + \text{h.c.}, \quad (11)$$

where S is a singlet scalar of the SM, which breaks the flavor symmetry spontaneously by a VEV $\langle S \rangle$. For simplicity, we assume $\Lambda_d = \Lambda_u \equiv \Lambda$ and $\langle S \rangle / \Lambda = \lambda$ to be like the Cabibbo angle. In non-supersymmetric models, powers of S^\dagger should be allowed. However, since this possibility is forbidden in the super-potential of the supersymmetric model, we ignore S^\dagger .

Let us define $U(1)$ charges for doublet quarks Q_i and singlets \bar{d}_i and \bar{u}_i :

$$(Q_1, Q_2, Q_3) = (a_1, a_2, a_3), \quad (\bar{d}_1, \bar{d}_2, \bar{d}_3) = (x_1, x_2, x_3), \quad (\bar{u}_1, \bar{u}_2, \bar{u}_3) = (r_1, r_2, r_3), \quad (12)$$

with $U(1)$ charges h_d , h_u and -1 for the Higgs H_d , H_u and S , respectively.

Then we get $m_{ij} = a_i + x_j + h_d$ and $n_{ij} = a_i + r_j + h_u$ ($i, j = 1, 2, 3$). If m_{ij} and n_{ij} are negative, those entries are zeros. These rules together with experimental constraints of eqs.(9) and (10) uniquely fix unknown entries of the quark mass matrix apart from coefficients of $\mathcal{O}(1)$ as follows:

$$M_d \simeq \lambda^3 m_0 \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix}, \quad (13)$$

where $\delta = 4$ and $\eta = 3$ are fixed in eq.(9), and

$$M_u \simeq m_0 \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^4 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix}, \quad (14)$$

where $\kappa = 2$ is fixed in eq.(10). The second and third columns in M_d are same due to $x_2 = x_3$. We find a few undesirable features in these mass matrices in the viewpoint of the quark mass matrix phenomenology. The first point is in (1-3) entries of M_d and M_u . The successful relation between the CKM matrix elements and quark masses [23]

$$\left| \frac{V_{ub}}{V_{cb}} \right| \simeq \sqrt{\frac{m_u}{m_c}}, \quad (15)$$

is derived by the assumption of zero texture in the (1-3) entry and the suppressed (3-2) entry. The $\mathcal{O}(\lambda^3)$ in the (1-3) entry, which is the same order of the (1-2) entry, spoils this relation considerably. This situation is easily understood by taking into account $V_{ub} = D_{13} + U_{21}^* D_{23} + U_{31}^*$, where D and U are left-handed unitary matrices which diagonalize mass matrices M_d and M_u , respectively. Mixings $D_{13} \sim \lambda^3$, $U_{21}^* D_{23} \sim \lambda^3$ and $U_{31}^* \sim \lambda^3$ are easily obtained from eqs.(13) and (14) [23]. Unless an accidental cancellation among D_{13} , $U_{21}^* D_{23}$ and U_{31}^* is realized, the experimentally consistent prediction $V_{ub} \sim \lambda^4$ is not expected. It may be useful to comment on the effect of the (3-2) entry. Even if the (1-3) entry is suppressed compared with λ^3 , $D_{13} \sim \lambda^3$ is still kept as far as the M_d matrix has such entries as $M_{d32} \simeq M_{d33}$ due to $M_{d12} \simeq \lambda^6$. The effect of the (3-2) entry is easily understood by the following formula:

$$D_{13} \sim \frac{M_{d13}}{M_{d33}} + \frac{M_{d12}M_{d32}}{M_{d33}^2} . \quad (16)$$

The second point is in the (1-2) entry in the up quark sector. The texture in eq.(14) gives a mixing $U_{12} = \mathcal{O}(\lambda)$ for the u-quark sector. This does not guarantee a successful formula of [24]

$$V_{us} \simeq \sqrt{\frac{m_d}{m_s}} , \quad (17)$$

because of the same order contribution from the up-quark sector. However, $U_{12} = \mathcal{O}(\lambda)$ may be harmless if a coefficient smaller than 1 and a phase are taken account.

Let us consider the lepton sector. The effective Yukawa couplings below the mass scale Λ are of the form

$$L_i \bar{\ell}_j H_d \left(\frac{S}{\Lambda} \right)^{m_{ij}} + \frac{1}{M_R} L_i L_j H_u H_u \left(\frac{S}{\Lambda} \right)^{n_{ij}} , \quad (18)$$

where M_R is a relevant high mass scale, and $U(1)$ charges for doublet leptons L_i and singlets $\bar{\ell}_i$ are defined as

$$(L_1, L_2, L_3) = (\alpha_1, \alpha_2, \alpha_3), \quad (\bar{\ell}_1, \bar{\ell}_2, \bar{\ell}_3) = (\rho_1, \rho_2, \rho_3) , \quad (19)$$

respectively. The charged lepton mass matrix is given apart from the constant mass factor as

$$M_\ell \sim \begin{pmatrix} \lambda^{\alpha_1+\rho_1} & \lambda^{\alpha_1+\rho_2} & \lambda^{\alpha_1+\rho_3} \\ \lambda^{\alpha_2+\rho_1} & \lambda^{\alpha_2+\rho_2} & \lambda^{\alpha_2+\rho_3} \\ \lambda^{\alpha_3+\rho_1} & \lambda^{\alpha_3+\rho_2} & \lambda^{\alpha_3+\rho_3} \end{pmatrix} , \quad (20)$$

and the effective left-handed Majorana neutrino mass matrix is

$$M_\nu \sim \begin{pmatrix} \lambda^{2\alpha_1} & \lambda^{\alpha_1+\alpha_2} & \lambda^{\alpha_1+\alpha_3} \\ \lambda^{\alpha_1+\alpha_2} & \lambda^{2\alpha_2} & \lambda^{\alpha_2+\alpha_3} \\ \lambda^{\alpha_1+\alpha_3} & \lambda^{\alpha_2+\alpha_3} & \lambda^{2\alpha_3} \end{pmatrix} , \quad (21)$$

where suppression factors due to the Higgs $U(1)$ charges h_d and h_u are omitted. Hereafter, we omit the constant mass terms. If the large mixing is required in the neutrino mixing matrix(MNS mixing matrix) [2], $U(1)$ charges are determined. The relation of $\alpha_2 = \alpha_3$ makes the democratic submatrix in the (2-3) sector of the neutrino mass

matrix. Then the large mixing is naturally obtained in the neutrino sector. What happens in the charged lepton sector? The (2- i) and (3- i) entries ($i = 1, 2, 3$) are same ones due to $\alpha_2 = \alpha_3$. In particular, the same value of (2-3) and (3-3) entries gives the large mixing angle in the left-handed charged lepton sector. Thus $\alpha_2 = \alpha_3$ essentially leads to large mixings in both neutrino sector and charged lepton sector. In the viewpoint of the flavor symmetry, this is an unpleasant feature because the neutrino mass hierarchy is determined only by unknown coefficients of $\mathcal{O}(1)$. Moreover the near-maximal MNS mixing is not guaranteed in the case of both large angle rotations. Magnitudes of mixing angles and phases in both sector must be tuned if the experimental MNS mixing in the (2-3) sector is the near-maximal mixing.

Another way to get the large mixing with avoiding this situation is to assume the relation $\alpha_2 = -\alpha_3 > 0$ [19]. Then the neutrino mass hierarchy turns to $m_3 \simeq m_2 \gg m_1$, which is contradict with the ones in eqs.(3) and (4). Thus the $U(1)$ flavor symmetry implies undesirable features in quark and lepton sectors. The situation is not improved even if more $U(1)$ flavor symmetries such as $U(1)_1 \times U(1)_2$ or $U(1)_1 \times U(1)_2 \times U(1)_3$ are introduced.

To suppress $|V_{ub}|$ below λ^3 , certain entries in mass matrices have to be suppressed relative to their naive values. This suppression is realized in the discrete symmetry Z_m , which is a subgroup of the $U(1)$ symmetry as shown in [22]. Anyway, in order to overcome these undesirable features of quark-lepton mass matrices, one should consider beyond the $U(1)$ flavor symmetry. We discuss the discrete symmetry Z_m for both quark and lepton mass matrices.

3 Quark-Lepton Mass Matrices with Z_m Symmetry

A single Z_m flavor symmetry is not helpful to improve above discussed undesirable features because certain entries in the mass matrix cannot be suppressed relative to

their naive values. Let us discuss the extended symmetry $U(1) \times Z_m$. The effective Yukawa couplings of the lepton sector are given by extending eq.(18) with new Higgs S_1 and S_2 as follows [22]:

$$L_i \bar{\ell}_j H_d \epsilon_1^{m_{ij}} \epsilon_2^{m'_{ij}} + \frac{1}{M_R} L_i L_j H_u H_u \epsilon_1^{n_{ij}} \epsilon_2^{n'_{ij}} , \quad (22)$$

where $\epsilon_1 \equiv < S_1 > / \Lambda$ and $\epsilon_2 \equiv < S_2 > / \Lambda$ are assumed to be expressed in terms of λ . The $U(1)$ and Z_m charges for doublet leptons L_i and singlets $\bar{\ell}_i$ are defined as

$$\begin{aligned} L_1(\alpha_1, \beta_1) , \quad L_2(\alpha_2, \beta_2) , \quad L_3(\alpha_3, \beta_3) , \\ \bar{\ell}_1(\rho_1, \sigma_1) , \quad \bar{\ell}_2(\rho_2, \sigma_2) , \quad \bar{\ell}_3(\rho_3, \sigma_3) , \end{aligned} \quad (23)$$

respectively. In order to see the neutrino mass hierarchy and the large mixing, we discuss the (2-3) submatrices of the charged lepton and the left-handed Majorana neutrino mass matrices:

$$M_\ell \sim \begin{pmatrix} \epsilon_1^{\alpha_2+\rho_2} \epsilon_2^{\beta_2+\sigma_2} & \epsilon_1^{\alpha_2+\rho_3} \epsilon_2^{\beta_2+\sigma_3} \\ \epsilon_1^{\alpha_3+\rho_2} \epsilon_2^{\beta_3+\sigma_2} & \epsilon_1^{\alpha_3+\rho_3} \epsilon_2^{\beta_3+\sigma_3} \end{pmatrix} , \quad (24)$$

and

$$M_\nu \sim \begin{pmatrix} \epsilon_1^{2\alpha_2} \epsilon_2^{2\beta_2} & \epsilon_1^{\alpha_2+\alpha_3} \epsilon_2^{\beta_2+\beta_3} \\ \epsilon_1^{\alpha_2+\alpha_3} \epsilon_2^{\beta_2+\beta_3} & \epsilon_1^{2\alpha_3} \epsilon_2^{2\beta_3} \end{pmatrix} , \quad (25)$$

respectively. If we take $\alpha_2 = \alpha_3$ for $U(1)$ charges and $\beta_1 = \beta_2$ for Z_m charges, both mixing angles of charged leptons and neutrinos could be large as discussed in eqs.(20) and (21). However only the mixing angle of the charged lepton could be large due to the Z_m symmetry if we put

$$\beta_2 + \sigma_3 = m , \quad \epsilon_1^{\alpha_2} = \epsilon_1^{\alpha_3} \epsilon_2^{\beta_3+\sigma_3} , \quad (26)$$

with $\beta_3 + \sigma_3 \neq 0$. Then the neutrino mass matrix has a hierarchical structure. By use of these two conditions, we get

$$M_\nu \sim \begin{pmatrix} \epsilon_2^{2m} & \epsilon_2^m \\ \epsilon_2^m & 1 \end{pmatrix} \epsilon_1^{2\alpha_3} \epsilon_2^{2\beta_3} . \quad (27)$$

By taking $\epsilon_2 = \lambda$, we get the neutrino mass ratio $m_2 : m_3 = \lambda^{2m} : 1$. As seen in eq.(5), the experimental data suggest $m = 1$, which is unfavorable for the Z_m symmetry. However, as shown by Grossman, Nir and Shadmi [18], if the condition $2\beta_2 = m$ is added to eq.(26), the neutrino mass matrix turns to

$$M_\nu \sim \begin{pmatrix} \epsilon_2^m & \epsilon_2^m \\ \epsilon_2^m & 1 \end{pmatrix} \epsilon_1^{2\alpha_3} \epsilon_2^{2\beta_3}, \quad (28)$$

which leads to $m_2 : m_3 = \lambda^m : 1$. It is remarked that the mass enhancement of m_2 is realized due to the Z_m symmetry [18]. Only the Z_2 symmetry is consistent with the experimental mass ratio of m_2 and m_3 .

After putting $U(1)$ charges $\alpha_3 = 0$, $\alpha_2 = 1$, $\rho_3 = 2$ and Z_2 charges $\beta_3 = 0$, $\beta_2 = 1$, $\sigma_3 = 1$, which satisfy above conditions, it is easily found that $\epsilon_1 = \lambda$ should be fixed as well as $\epsilon_2 = \lambda$ to get $m_2 : m_3 = \lambda^2 : 1$ in the neutrino mass matrix. Therefore we take $\epsilon_1 = \epsilon_2 = \lambda$ in following analyses.

Let us study the quark sector in the $U(1) \times Z_2$ symmetry. The effective Yukawa couplings of the quark sector are of the form

$$Q_i \bar{d}_j H_d \epsilon_1^{m_{ij}} \epsilon_2^{m'_{ij}} + Q_i \bar{u}_j H_u \epsilon_1^{n_{ij}} \epsilon_2^{n'_{ij}}. \quad (29)$$

The $U(1)$ and Z_2 charges for doublet quarks Q_i , singlets \bar{d}_i and \bar{u}_i are defined as

$$\begin{aligned} & Q_1(a_1, b_1), \quad Q_2(a_2, b_2), \quad Q_3(a_3, b_3), \\ & \bar{d}_1(x_1, y_1), \quad \bar{d}_2(x_2, y_2), \quad \bar{d}_3(x_3, y_3), \\ & \bar{u}_1(r_1, s_1), \quad \bar{u}_2(r_2, s_2), \quad \bar{u}_3(r_3, s_3), \end{aligned} \quad (30)$$

respectively. Then we can express quark mass matrices in terms of $U(1)$ and Z_2 charges apart from order one coefficients:

$$M_d \sim \begin{pmatrix} \epsilon_1^{a_1+x_1} \epsilon_2^{b_1+y_1} & \epsilon_1^{a_1+x_2} \epsilon_2^{b_1+y_2} & \epsilon_1^{a_1+x_3} \epsilon_2^{b_1+y_3} \\ \epsilon_1^{a_2+x_1} \epsilon_2^{b_2+y_1} & \epsilon_1^{a_2+x_2} \epsilon_2^{b_2+y_2} & \epsilon_1^{a_2+x_3} \epsilon_2^{b_2+y_3} \\ \epsilon_1^{x_1} \epsilon_2^{y_1} & \epsilon_1^{x_2} \epsilon_2^{y_2} & \epsilon_1^{x_3} \epsilon_2^{y_3} \end{pmatrix}, \quad (31)$$

and

$$M_u \sim \begin{pmatrix} \epsilon_1^{a_1+r_1} \epsilon_2^{b_1+s_1} & \epsilon_1^{a_1+r_2} \epsilon_2^{b_1+s_2} & \epsilon_1^{a_1+r_3} \epsilon_2^{b_1+s_3} \\ \epsilon_1^{a_2+r_1} \epsilon_2^{b_2+s_1} & \epsilon_1^{a_2+r_2} \epsilon_2^{b_2+s_2} & \epsilon_1^{a_2+r_3} \epsilon_2^{b_2+s_3} \\ \epsilon_1^{r_1} \epsilon_2^{s_1} & \epsilon_1^{r_2} \epsilon_2^{s_2} & \epsilon_1^{r_3} \epsilon_2^{s_3} \end{pmatrix}, \quad (32)$$

where a_3 and b_3 are set to be zero without loss of generality.

In order to avoid a phenomenological defect $D_{13} \sim \lambda^3$, we search $U(1)$ and Z_2 charges leading to $M_{d32} \sim \lambda^5$ and $M_{d13} \leq \lambda^7$ under four conditions

$$M_{d33} = \lambda^3, \quad M_{d23} = \lambda^5, \quad M_{d22} = \lambda^5, \quad M_{d12} = \lambda^6. \quad (33)$$

We get only two solutions to satisfy these four conditions and $M_{d32} \sim \lambda^5$ as

$$\begin{aligned} (x_3 = 3, \quad y_3 = 0, \quad x_2 = 4, \quad y_2 = 1, \quad a_2 = 1, \quad b_2 = 1, \quad a_1 = 1, \quad b_1 = 0), \\ (x_3 = 3, \quad y_3 = 0, \quad x_2 = 4, \quad y_2 = 1, \quad a_2 = 1, \quad b_2 = 1, \quad a_1 = 2, \quad b_1 = 1). \end{aligned} \quad (34)$$

However we get $M_{d13} = \lambda^4$ and $M_{d13} = \lambda^6$ for each solution, respectively. Thus $M_{d13} \leq \lambda^7$ cannot be obtained in the framework of the $U(1) \times Z_2$ flavor symmetry. In conclusion, the defect of V_{ub} is still kept although the defect in the lepton sector is removed.

If the Z_m symmetry with $m \geq 3$ is introduced, the problem of V_{ub} could be resolved as shown in ref.[22]. However the neutrino mass ratio in eq.(5) allows only $m = 2$. In order to remove the defect of V_{ub} , we should proceed to the minimal extension $U(1) \times Z_2 \times Z'_2$ flavor symmetry.²

4 Mass matrices with $U(1) \times Z_2 \times Z'_2$ Symmetry

In the $U(1) \times Z_2 \times Z'_2$ symmetry, the effective Yukawa couplings of the lepton sector are given by extending eq.(18) with three Higgs S_1 , S_2 and S_3 as follows:

$$L_i \bar{\ell}_j H_d \epsilon_1^{m_{ij}} \epsilon_2^{m'_{ij}} \epsilon_3^{m''_{ij}} + \frac{1}{M_R} L_i L_j H_u H_u \epsilon_1^{n_{ij}} \epsilon_2^{n'_{ij}} \epsilon_3^{n''_{ij}}, \quad (35)$$

²We distinguish two Z_2 symmetries by using prime.

where a new parameter $\epsilon_3 \equiv < S_3 > / \Lambda$ is introduced. The $U(1)$, Z_2 and Z'_2 charges for doublet leptons L_i and singlets $\bar{\ell}_i$ are:

$$\begin{aligned} L_1(\alpha_1, \beta_1, \gamma_1) , \quad L_2(\alpha_2, \beta_2, \gamma_2) , \quad L_3(\alpha_3, \beta_3, \gamma_3) , \\ \bar{\ell}_1(\rho_1, \sigma_1, \tau_1) , \quad \bar{\ell}_2(\rho_2, \sigma_2, \tau_2) , \quad \bar{\ell}_3(\rho_3, \sigma_3, \tau_3) , \end{aligned} \quad (36)$$

respectively. The charged lepton and the neutrino mass matrices are given as follows:

$$M_\ell \sim \begin{pmatrix} \epsilon_1^{\alpha_1+\rho_1} \epsilon_2^{\beta_1+\sigma_1} \epsilon_3^{\gamma_1+\tau_1} & \epsilon_1^{\alpha_1+\rho_2} \epsilon_2^{\beta_1+\sigma_2} \epsilon_3^{\gamma_1+\tau_2} & \epsilon_1^{\alpha_1+\rho_3} \epsilon_2^{\beta_1+\sigma_3} \epsilon_3^{\gamma_1+\tau_3} \\ \epsilon_1^{\alpha_2+\rho_1} \epsilon_2^{\beta_2+\sigma_1} \epsilon_3^{\gamma_2+\tau_1} & \epsilon_1^{\alpha_2+\rho_2} \epsilon_2^{\beta_2+\sigma_2} \epsilon_3^{\gamma_2+\tau_2} & \epsilon_1^{\alpha_2+\rho_3} \epsilon_2^{\beta_2+\sigma_3} \epsilon_3^{\gamma_2+\tau_3} \\ \epsilon_1^{\alpha_3+\rho_1} \epsilon_2^{\beta_3+\sigma_1} \epsilon_3^{\gamma_3+\tau_1} & \epsilon_1^{\alpha_3+\rho_2} \epsilon_2^{\beta_3+\sigma_2} \epsilon_3^{\gamma_3+\tau_2} & \epsilon_1^{\alpha_3+\rho_3} \epsilon_2^{\beta_3+\sigma_3} \epsilon_3^{\gamma_3+\tau_3} \end{pmatrix} , \quad (37)$$

and

$$M_\nu \sim \begin{pmatrix} \epsilon_1^{2\alpha_1} \epsilon_2^{2\beta_1} \epsilon_3^{2\gamma_1} & \epsilon_1^{\alpha_1+\alpha_2} \epsilon_2^{\beta_1+\beta_2} \epsilon_3^{\gamma_1+\gamma_2} & \epsilon_1^{\alpha_1+\alpha_3} \epsilon_2^{\beta_1+\beta_3} \epsilon_3^{\gamma_1+\gamma_3} \\ \epsilon_1^{\alpha_2+\alpha_1} \epsilon_2^{\beta_2+\beta_1} \epsilon_3^{\gamma_2+\gamma_1} & \epsilon_1^{2\alpha_2} \epsilon_2^{2\beta_2} \epsilon_3^{2\gamma_2} & \epsilon_1^{\alpha_2+\alpha_3} \epsilon_2^{\beta_2+\beta_3} \epsilon_3^{\gamma_2+\gamma_3} \\ \epsilon_1^{\alpha_3+\alpha_1} \epsilon_2^{\beta_3+\beta_1} \epsilon_3^{\gamma_3+\gamma_1} & \epsilon_1^{\alpha_3+\alpha_2} \epsilon_2^{\beta_3+\beta_2} \epsilon_3^{\gamma_3+\gamma_2} & \epsilon_1^{2\alpha_3} \epsilon_2^{2\beta_3} \epsilon_3^{2\gamma_3} \end{pmatrix} , \quad (38)$$

where we can take $\alpha_3 = 0$, $\beta_3 = 0$ and $\gamma_3 = 0$ without loss of generality. In order to get the large mixing angle of the (2-3) sector in the charged lepton mass matrix keeping the small mixing angle in the neutrino mass matrix, we take similar conditions as in eq.(26), either

$$\beta_2 + \sigma_3 = 2 , \quad \epsilon_1^{\alpha_2} \epsilon_3^{\gamma_2+\tau_3} = \epsilon_2^{\sigma_3} \epsilon_3^{\tau_3} , \quad (39)$$

or

$$\gamma_2 + \tau_3 = 2 , \quad \epsilon_1^{\alpha_2} \epsilon_2^{\beta_2+\sigma_3} = \epsilon_2^{\sigma_3} \epsilon_3^{\tau_3} . \quad (40)$$

Then the neutrino mass matrix has a hierarchy structure. By taking either $\epsilon_1 = \lambda$, $\epsilon_2 = \lambda$ and $\alpha_2 = 1$ in addition to eq.(39) or $\epsilon_1 = \lambda$, $\epsilon_3 = \lambda$ and $\alpha_2 = 1$ in addition to eq.(40), we get the neutrino mass ratio $m_2 : m_3 = \lambda^2 : 1$. In the first(second) case, $\epsilon_3(\epsilon_2)$ is not necessary to be λ . However we take $\epsilon_1 = \epsilon_2 = \epsilon_3 = \lambda$ for simplicity. We can fix α_2 , β_2 , γ_2 and $\sigma_3(\tau_3)$ by the condition eq.(39)(eq.(40)) and the neutrino mass

hierarchy as follows:

$$\begin{aligned} (\alpha_2 = 1, \quad \beta_2 = 1, \quad \gamma_2 = 0, \quad \sigma_3 = 1), \\ (\alpha_2 = 1, \quad \beta_2 = 0, \quad \gamma_2 = 1, \quad \tau_3 = 1). \end{aligned} \quad (41)$$

Since two solutions are equivalent after interchange of ϵ_2 and ϵ_3 , we consider only the solution of the first case hereafter. The condition $M_{\ell 33} = \lambda^3$ gives two set of (ρ_3, τ_3)

$$(\rho_3 = 2, \quad \tau_3 = 0), \quad \text{or} \quad (\rho_3 = 1, \quad \tau_3 = 1). \quad (42)$$

Furthermore conditions $M_{\ell 22} = \lambda^5$ and $M_{\ell 32} = \lambda^5$ fix

$$(1) (\rho_2 = 4, \sigma_2 = 1, \tau_2 = 0), \quad (2) (\rho_2 = 3, \sigma_2 = 1, \tau_2 = 1). \quad (43)$$

At the last step, we can fix charges by using conditions $M_{\ell 11} \sim \lambda^7$, $M_{\ell 12} \leq \lambda^7$ and $M_{\ell 21} M_{\ell 12} \leq \lambda^{12}$. For the solution (1) in eq.(43) we get two sets of charges:

$$(A) (\alpha_1 = 1, \beta_1 = 0, \gamma_1 = 1), \quad (\rho_1 = 4, \sigma_1 = 1, \tau_1 = 0), \quad (44)$$

$$(B) (\alpha_1 = 1, \beta_1 = 0, \gamma_1 = 1), \quad (\rho_1 = 5, \sigma_1 = 0, \tau_1 = 0), \quad (45)$$

and for the solution (2):

$$(C) (\alpha_1 = 2, \beta_1 = 0, \gamma_1 = 0), \quad (\rho_1 = 4, \sigma_1 = 1, \tau_1 = 0), \quad (46)$$

$$(D) (\alpha_1 = 2, \beta_1 = 0, \gamma_1 = 0), \quad (\rho_1 = 5, \sigma_1 = 0, \tau_1 = 0). \quad (47)$$

For cases (A) and (B), $(\rho_3 = 1, \tau_3 = 1)$ in eq.(42) leads to $M_{\ell 13} \sim \lambda^3$, which gives the democratic mixing for left-handed charged leptons. For other cases, $M_{\ell 13} \sim \lambda^5$ is predicted.

We show charged lepton mass matrices and neutrino ones for each case:

$$(A) M_\ell \sim \begin{pmatrix} \lambda^7 & \lambda^7 & \lambda^{5(3)} \\ \lambda^5 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^5 & \lambda^3 \end{pmatrix}, \quad M_\nu \sim \begin{pmatrix} \lambda^2 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & \lambda^2 & 1 \end{pmatrix}, \quad (48)$$

$$(B) \quad M_\ell \sim \begin{pmatrix} \lambda^7 & \lambda^7 & \lambda^{5(3)} \\ \lambda^7 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^5 & \lambda^3 \end{pmatrix}, \quad M_\nu \sim \begin{pmatrix} \lambda^2 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & \lambda^2 & 1 \end{pmatrix}, \quad (49)$$

$$(C) \quad M_\ell \sim \begin{pmatrix} \lambda^7 & \lambda^7 & \lambda^5 \\ \lambda^5 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^5 & \lambda^3 \end{pmatrix}, \quad M_\nu \sim \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & \lambda^2 & 1 \end{pmatrix}, \quad (50)$$

$$(D) \quad M_\ell \sim \begin{pmatrix} \lambda^7 & \lambda^7 & \lambda^5 \\ \lambda^7 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^5 & \lambda^3 \end{pmatrix}, \quad M_\nu \sim \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & \lambda^2 & 1 \end{pmatrix}. \quad (51)$$

The neutrino mass hierarchy is $m_3 : m_2 : m_1 = 1 : \lambda^2 : \lambda^2$ for cases (A) and (B), and $m_3 : m_2 : m_1 = 1 : \lambda^2 : \lambda^4$ for cases (C) and (D). In these four solutions, we expect neutrino mixings $U_{e2} \sim \lambda^2$ and $U_{e3} \sim \lambda^2$ as well as $U_{\mu 2} \simeq 1/\sqrt{2}$. It may be useful to comment on that U_{e2} could be also large for cases (A) and (B) if coefficients of $M_{\nu 11}$ and $M_{\nu 22}$ close each other. However we do not expect such a difference of $\mathcal{O}(\lambda^4)$ between $M_{\nu 11}$ and $M_{\nu 22}$. In these cases, $M_{\ell 13} \simeq \lambda^3$ also provides the possibility to give a large U_{e2} .

Let us discuss quark mass matrices in the $U(1) \times Z_2 \times Z'_2$ flavor symmetry. The effective Yukawa couplings of the quark sector are of the form

$$Q_i \bar{d}_j H_d \epsilon_1^{m_{ij}} \epsilon_2^{m'_{ij}} \epsilon_3^{m''_{ij}} + Q_i \bar{u}_j H_u \epsilon_1^{n_{ij}} \epsilon_2^{n'_{ij}} \epsilon_3^{n''_{ij}}. \quad (52)$$

The $U(1)$, Z_2 and Z'_2 charges for doublet quarks Q_i , singlets \bar{d}_i and \bar{u}_i are:

$$\begin{aligned} & Q_1(a_1, b_1, c_1), \quad Q_2(a_2, b_2, c_2), \quad Q_3(a_3, b_3, c_3), \\ & \bar{d}_1(x_1, y_1, z_1), \quad \bar{d}_2(x_2, y_2, z_2), \quad \bar{d}_3(x_3, y_3, z_3), \\ & \bar{u}_1(r_1, s_1, t_1), \quad \bar{u}_2(r_2, s_2, t_2), \quad \bar{u}_3(r_3, s_3, t_3), \end{aligned} \quad (53)$$

respectively. Then quark mass matrices are written as

$$M_d \sim \begin{pmatrix} \epsilon_1^{a_1+x_1} \epsilon_2^{b_1+y_1} \epsilon_3^{c_1+z_1} & \epsilon_1^{a_1+x_2} \epsilon_2^{b_1+y_2} \epsilon_3^{c_1+z_2} & \epsilon_1^{a_1+x_3} \epsilon_2^{b_1+y_3} \epsilon_3^{c_1+z_3} \\ \epsilon_1^{a_2+x_1} \epsilon_2^{b_2+y_1} \epsilon_3^{c_2+z_1} & \epsilon_1^{a_2+x_2} \epsilon_2^{b_2+y_2} \epsilon_3^{c_2+z_2} & \epsilon_1^{a_2+x_3} \epsilon_2^{b_2+y_3} \epsilon_3^{c_2+z_3} \\ \epsilon_1^{x_1} \epsilon_2^{y_1} \epsilon_3^{z_1} & \epsilon_1^{x_2} \epsilon_2^{y_2} \epsilon_3^{z_2} & \epsilon_1^{x_3} \epsilon_2^{y_3} \epsilon_3^{z_3} \end{pmatrix}, \quad (54)$$

$$M_u \sim \begin{pmatrix} \epsilon_1^{a_1+r_1} \epsilon_2^{b_1+s_1} \epsilon_3^{c_1+t_1} & \epsilon_1^{a_1+r_2} \epsilon_2^{b_1+s_2} \epsilon_3^{c_1+t_2} & \epsilon_1^{a_1+r_3} \epsilon_2^{b_1+s_3} \epsilon_3^{c_1+t_3} \\ \epsilon_1^{a_2+r_1} \epsilon_2^{b_2+s_1} \epsilon_3^{c_2+t_1} & \epsilon_1^{a_2+r_2} \epsilon_2^{b_2+s_2} \epsilon_3^{c_2+t_2} & \epsilon_1^{a_2+r_3} \epsilon_2^{b_2+s_3} \epsilon_3^{c_2+t_3} \\ \epsilon_1^{r_1} \epsilon_2^{s_1} \epsilon_3^{t_1} & \epsilon_1^{r_2} \epsilon_2^{s_2} \epsilon_3^{t_2} & 1 \end{pmatrix}, \quad (55)$$

where we take $a_3 = b_3 = c_3 = 0$ and $r_3 = s_3 = t_3 = 0$ without loss of generality. As shown in the previous section, relevant charges are obtained under four conditions in eq.(33). At first, we search $U(1)$ and Z_2 charges leading to $M_{d32} \sim \lambda^5$ and $M_{d33} \sim \lambda^3$. The unique solution is

$$(x_3 = 3, y_3 = 0, z_3 = 0), \quad (x_2 = 3, y_2 = 1, z_1 = 1), \quad (56)$$

because $x_3 \leq 2$ and $x_2 \geq 4$ always leads to $M_{d12} \leq M_{d13}$, which is not consistent with the observed CKM matrix. Next, we get two sets

$$(a_2 = 1, b_2 = 1, c_2 = 0), \quad (a_2 = 1, b_2 = 0, c_2 = 1), \quad (57)$$

by using conditions $M_{d22} \sim \lambda^5$ and $M_{d23} \sim \lambda^5$. Now our concerned elements M_{d12} and M_{d13} are

$$M_{d12} = \epsilon_1^{a_1+3} \epsilon_2^{b_1+1} \epsilon_3^{c_1+1}, \quad M_{d13} = \epsilon_1^{a_1+3} \epsilon_2^{b_1} \epsilon_3^{c_1}. \quad (58)$$

By taking

$$a_1 = 3, \quad b_1 = 1, \quad c_1 = 1, \quad (59)$$

we get desirable elements $M_{d12} = \lambda^6$ and $M_{d13} = \lambda^8$. This is due to the $Z_2 \times Z'_2$ symmetry as seen in eq.(58). At the last step, the down quark mass ratio of m_d and m_s gives

$$x_1 = 4, \quad y_1 = 1, \quad c_1 = 1. \quad (60)$$

The down quark mass matrix is uniquely determined as follows:

$$M_d \sim \begin{pmatrix} \lambda^7 & \lambda^6 & \lambda^8 \\ \lambda^6 & \lambda^5 & \lambda^5 \\ \lambda^6 & \lambda^5 & \lambda^3 \end{pmatrix}. \quad (61)$$

Let us consider the up quark sector. By the condition $M_{u22} = \lambda^4$, we get

$$(1) \ r_2 = 2, \ s_2 = 0, \ t_2 = 0, \quad (2) \ r_2 = 2, \ s_2 = 1, \ t_2 = 1, \quad (3) \ r_2 = 3, \ s_2 = 1, \ t_2 = 0. \quad (62)$$

Each solution gives

$$(1) M_{u12} = \lambda^7, M_{u32} = \lambda^2, \quad (2) M_{u12} = \lambda^5, M_{u32} = \lambda^4, \quad (3) M_{u12} = \lambda^7, M_{u32} = \lambda^4, \quad (63)$$

respectively. Since solutions (1) and (3) in eq.(63) lead to rather small V_{ub} ($\simeq \mathcal{O}(\lambda^5)$), we choose the solution (2).

The condition $M_{u11} = \lambda^8$ determines (r_1, s_1, t_1) parameters as follows:

$$(1) r_1 = 5, s_1 = 1, t_1 = 1, \quad (2) r_1 = 4, s_1 = 0, t_1 = 1, \\ (3) r_1 = 4, s_1 = 1, t_1 = 0. \quad (4) r_1 = 3, s_1 = 0, t_1 = 0, \quad (64)$$

which lead to

$$(1) M_{u21} = \lambda^7, M_{u31} = \lambda^7, \quad (2) M_{u21} = \lambda^7, M_{u31} = \lambda^5, \\ (3) M_{u21} = \lambda^5, M_{u31} = \lambda^5, \quad (4) M_{u21} = \lambda^5, M_{u31} = \lambda^3, \quad (65)$$

respectively. The solutions (3) and (4) are excluded because those give the large m_u due to $M_{u12} \simeq M_{u21} \simeq \lambda^5$. In conclusion, the up quark mass matrix is given as follows:

$$M_u \sim \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^5 \\ \lambda^7 & \lambda^4 & \lambda^2 \\ \lambda^{7(5)} & \lambda^2 & 1 \end{pmatrix}. \quad (66)$$

In our quark mass matrices, (1-3) entries are suppressed in both down and up quark sectors due to the additional $Z_2 \times Z_2'$ symmetry. Now let consider the ratio V_{ub}/V_{cb} in our mass matrices. Since $V_{cb} \simeq D_{23} + U_{32}^*$ and $V_{ub} \simeq D_{13} + U_{21}^* D_{23} + U_{31}^*$, we can express the ratio as

$$\left| \frac{V_{ub}}{V_{cb}} \right| \simeq \left| \frac{U_{12}^* (U_{23}^* - D_{23})}{D_{23} - U_{23}^*} \right| = |U_{12}|, \quad (67)$$

where we used $U_{31} \simeq U_{12} U_{23} - U_{13}$, $U_{21} \simeq -U_{12}$ and $U_{32} \simeq -U_{23}$, and D_{13} and U_{13} are neglected. Thus the V_{ub}/V_{cb} ratio depends on only $|U_{12}|$. Since we have $M_{u12} = \lambda^5$, which leads to $U_{12} = \mathcal{O}(\lambda)$, the V_{ub}/V_{cb} ratio is $a\lambda$, where a is a coefficient of $\mathcal{O}(1)$. If a is smaller than $1/2$, our model is consistent with the experimental value in eq.(8).

The element $M_{u12} = \lambda^6$, which leads to $U_{12} = \mathcal{O}(\lambda^2)$, may be favored in order to give $V_{ub}/V_{cb} \simeq \sqrt{m_u/m_c}$. We cannot get $M_{u12} \simeq \lambda^6$ in the $Z_2 \times Z'_2$ symmetry. However, since we have $V_{cb} = b\lambda^2$, where a coefficient b should be fixed to be $0.6 - 0.7$ with taking $\lambda = 0.22$ by the experimental value in eq.(8), we can express $V_{ub} \simeq U_{12}V_{cb} = ab\lambda^3$. Again if $a \leq 1/2$ with $b = 0.6 - 0.7$, $V_{ub} \sim \lambda^4$ is easily attainable. Thus, $U_{12} = \mathcal{O}(\lambda)$ is harmless as far as D_{13} and U_{13} are suppressed.

In our model, we assumed $\epsilon_1 = \epsilon_2 = \epsilon_3 = \lambda$. Even if we take another choice such as $\epsilon_1 = \epsilon_2 = \lambda$ and $\epsilon_3 = \lambda^2$, our conclusion is not so changed.

5 Summary

We have discussed phenomenological defects of $U(1)$ symmetric quark-lepton mass matrices. In the quark sector, the mixing V_{ub} is predicted to be large compared with the expected value λ^4 at the GUT scale. In the lepton sector, the same $U(1)$ charge such as $\alpha_2 = \alpha_3$ is required to give the large mixing in the neutrino sector, however the same charge also leads to the large mixing in the charged lepton sector. In the viewpoint of the flavor symmetry, this is an unpleasant feature because the neutrino mass hierarchy is determined only by unknown coefficients of $\mathcal{O}(1)$, and the near-maximal MNS mixing is not guaranteed in the case of both large angle rotations.

To suppress $|V_{ub}|$ below λ^3 , certain entries in the mass matrix have to be suppressed relative to their naive values. This suppression is realized in the additional discrete symmetry $Z_2 \times Z'_2$. This symmetry also leads to the hierarchical structure of the neutrino mass matrix while the large mixing is kept in the charged lepton sector. Moreover there is the enhancement of the neutrino mass m_2 , which is consistent with the experimental neutrino mass ratio obtained by atmospheric neutrinos and solar neutrinos. Asking the origin of the $Z_2 \times Z'_2$ symmetry is a subject for future investigations.

Acknowledgements

The author would like to express his thanks to the theory group of CFIF/IST in Portugal for their hospitality. This research is supported by the Grant-in-Aid for Science Research, Ministry of Education, Science and Culture, Japan(No.10140218, No.10640274).

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